

In Situ Imaging of Cold Atomic Gases

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Abstract: In general, the complex atomic susceptibility, that dictates both the amplitude and phase modulation imparted by an atom on a probing monochromatic light source, is highly dependent upon the local magnetic environment of the atom. While it is of great interest to study the dynamics of cold and ultra-cold atomic gases in strongly confining magnetic traps, the spatially dependent, and often quite complex, vector fields within these traps makes direct imaging difficult. Several novel techniques have been developed and implemented within the past two decades to image dense atomic clouds, not only in situ, but also non-destructively. In this letter we present two of these methods (dark-ground imaging and phase contrast imaging) and propose a new, third method that is designed to image few-atom clouds such as those found in modern, chip-based microtraps.

Modern experiments conducted on cold and ultra-cold atomic gases, be they bosonic or fermionic, rely heavily on probing the density profile of the system via the light-matter interaction. Traditional imaging techniques require each atom in the system be placed in a known, constant magnetic field; this requirement is often at odds with the preference that the system be probed *in situ*. Advanced imaging techniques were developed shortly after the first observations of Bose-Einstein condensation (BEC) that enabled the direct, non-destructive imaging of dense atomic clouds *in situ* [1,2]. These techniques, however, were developed to address the problem of very dense atomic clouds with optical densities much greater than 1 (typically >100) and are ill-suited to the low atom numbers typically found in modern chip-based microtraps. In general, the atomic scattering cross section and, by extension, the complex refractive index within a gas of neutral atoms depends upon both the magnitude and direction of the local magnetic field [3]. It is the goal of *in situ* imaging to remove this dependence, resulting in a constant effective cross section for all atoms within a magnetic trap.

ATOMIC ABSORPTION IMAGING

A laser of initial intensity I_0 passing through a general absorbing medium can be described by the equation:

$$\frac{dI(\vec{r}, t)}{dz} = -\alpha(\vec{r}, t) I(\vec{r}, t)$$

where α is the absorption coefficient and, in general, can be a function of both space and time. In writing down this equation we have assumed neither the intensity I_0 nor the absorption coefficient fluctuate on the time scale d/c , where d is the size of the absorbing medium and c is the

speed of light in the medium. For atomic gases it is often convenient to write α as a product of the atomic density and the atomic scattering cross section for a single atom [4].

$$\frac{dI(\vec{r}, t)}{dz} = -\eta(\vec{r})\sigma(\vec{r}, t) I(\vec{r}, t)$$

The only assumptions we have made here are that the absorption coefficient scales linearly with density and that the density is in steady state, no requirements are placed on the cross section. The solution to this equation gives us the intensity at a detector I_d :

$$I_d(t) = I_0 \text{Exp} \left[- \int_{-\infty}^{\infty} \eta(\vec{r})\sigma(\vec{r}, t) dz \right]$$

The signal recorded by a pixel on a camera with shutter time τ is proportional to the integral of the intensity at the pixel over the time τ . It is possible to measure the average transmission coefficient over the time τ by dividing the pixel value measurement of the amplitude modulated beam described above by the pixel value measurement of the unattenuated beam (i.e. the value with $\eta=0$)

$$\begin{aligned} T &\equiv \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} I_d dt / \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} I_0 dt \\ &= \frac{1}{\langle I_0 \rangle \tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} I_0 \text{Exp} \left[- \int_{-\infty}^{\infty} \eta(\vec{r})\sigma(\vec{r}, t) dz \right] dt \end{aligned}$$

Where $\langle I_0 \rangle$ is the time average of I_0 over the integration time τ . If the incoming intensity I_0 is fairly constant we can replace I_0 with $\langle I_0 \rangle$ and we find:

$$T = \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \text{Exp} \left[- \int_{-\infty}^{\infty} \eta(\vec{r}) \sigma(\vec{r}, t) dz \right] dt$$

To proceed past this point we must know more about the atomic scattering cross section σ . We know that the number of photons scattered per unit time (R_{sc}) is the product of the scattering cross section and the incoming photon flux per unit time, by definition[5].

$$R_{sc} = \frac{\sigma I}{\hbar \omega}$$

where I is the intensity of the incident light and ω is the angular frequency of the light. Using a master equation approach to model spontaneous emission we can also describe the scattering rate as:

$$R_{sc} = \Gamma \rho_{ee}$$

where ρ_{ee} is the atomic population in the excited state and Γ is the spontaneous decay rate [3,4,5]. Making the rotating wave approximation we can write down the optical Bloch equations in the usual way and find the steady state excited state population to be:

$$\rho_{ee}(t = \infty) = \frac{(\Omega/\Gamma)^2}{1 + 4(\Delta/\Gamma)^2 + 2(\Omega/\Gamma)^2}$$

$$\Delta \equiv \omega_{Laser} - \omega_{Transition}$$

Here Ω is the Rabi frequency and Δ is the full detuning between the incoming, monochromatic light and atomic transition energy over \hbar . Noting that Ω^2 is proportional to the intensity I , we can define a constant I_{sat} such that $2(\Omega/\Gamma)^2 = I/I_{sat}$ and write the scattering rate in steady state as:

$$R_{sc} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + 4(\Delta/\Gamma)^2 + I/I_{sat}}$$

$$I_{sat} \equiv \frac{c \epsilon_0 \Gamma^2 \hbar^2}{4 |\hat{\epsilon} \cdot \vec{d}|^2}$$

Here $\hat{\epsilon}$ is the polarization of the light and \vec{d} is the off diagonal dipole matrix element connecting the ground and excited states.

Comparing the two expressions for the scattering rate we find:

$$\frac{\sigma I}{\hbar \omega} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + 4(\Delta/\Gamma)^2 + I/I_{sat}}$$

Eliminating intensity and collecting the constants gives us a usable expression for the optical scattering cross section as a function of the laser parameters:

$$\sigma = \frac{\sigma_0}{1 + 4(\Delta/\Gamma)^2 + I/I_{sat}}$$

$$\sigma_0 \equiv \frac{\hbar \omega \Gamma}{2 I_{sat}} = \frac{2k}{\epsilon_0 \Gamma \hbar} |\hat{\epsilon} \cdot \vec{d}|^2$$

Plugging this into the expression for the time averaged transmission in the limit of low intensity, $I \ll I_{sat}$.

$$T \approx \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \text{Exp} \left[- \int_{-\infty}^{\infty} \frac{\sigma_0 \eta(\vec{r})}{1 + 4(\Delta(\vec{r}, t)/\Gamma)^2} dz \right] dt$$

The dependence of σ on space and time is now apparent and comes from spatially and temporally varying laser frequencies and atomic transition energies. If we fix the laser frequency and place our atomic cloud in a constant, known magnetic field the transmission simplifies to:

$$\text{Ln}[T(x, y)] = - \frac{\sigma_0}{1 + 4 \left(\frac{\Delta}{\Gamma} \right)^2} \eta_{col}(x, y)$$

$$\eta_{col}(x, y) \equiv \int_{-\infty}^{\infty} \eta(\vec{r}) dz$$

It is clear now why imaging an atomic cloud of unknown atomic density $\eta(\vec{r})$ *in situ* is quite difficult; even if a precise and analytic form of the magnetic fields within the trap exists and is time invariant, one still has to integrate

$$\text{Ln}[T(x, y)] = - \sigma_0 \int_{-\infty}^{\infty} \frac{\eta(\vec{r})}{1 + 4(\Delta(\vec{r})/\Gamma)^2} dz$$

In the following sections we will show how it is possible to remove the detuning's dependence on space in the limit of high atomic densities, such as those found in traditional BEC experiments, and in the limit of low atomic densities as found in more contemporary microtraps.

HIGH ATOMIC DENSITY *IN SITU* IMAGING

In cold, dense gases of neutral atoms, the optical density is high enough that the atomic cloud perturbs a laser detuned as far as 100 times the natural line width (Γ). As a function of laser detuning (Δ) the absorption cross section falls off as Δ^{-2} , while the refracted index only goes as Δ^{-1} [3], making it possible to image in a regime where amplitude modulation is negligible but phase modulation is not. In this way, a far detuned laser facilitates imaging that is not only *in situ*, as Zeeman shifting negligible when compared to $\sim 100\Gamma$, but also non-destructive.

In the limit of small phase shifts (ϕ), the phase modulated light leaving the atom cloud can be decomposed into the original plane wave and a small secondary plane wave that is $\pi/2$ out of phase. At the detector, we can write the complex electric field produced by the incoming plane wave as:

$$E(x, y, t) = |E_0(x, y)|e^{i\omega t}$$

The perturbed wave E_p can therefore be written as a function of the position dependent phase shift.

$$\begin{aligned} \frac{E_p(x, y, t)}{|E_0(x, y)|} &= e^{i[\omega t + \phi]} \\ &= \text{Cos}[\omega t + \phi] + i \text{Sin}[\omega t + \phi] \\ &= \text{Cos}[\omega t]\text{Cos}[\phi] - \text{Sin}[\omega t]\text{Sin}[\phi] \\ &\quad + i(\text{Sin}[\omega t]\text{Cos}[\phi] + \text{Cos}[\omega t]\text{Sin}[\phi]) \\ &\approx \text{Cos}[\omega t] + i\text{Sin}[\omega t] \\ &\quad + \phi(-\text{Sin}[\omega t] + i\text{Cos}[\omega t]) \\ &= e^{i\omega t} + \phi(x, y)e^{i(\omega t + \frac{\pi}{2})} \end{aligned}$$

This tells us that if the cloud is directly imaged onto a camera, via the setup illustrated in FIG.1, the measured intensity will be unchanged and we will see no effect from the atoms.

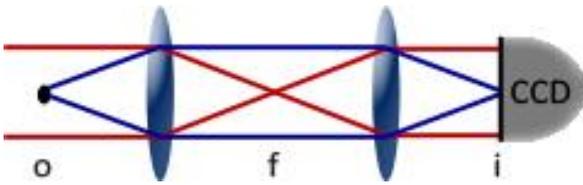


FIG.1 Ray trace of an incoming, far detuned laser (red) weakly interacting with a dense atomic cloud at position O. The atoms at O add a small, density dependent phase lag that can be approximated as a second, much weaker plane wave (blue) emitted from the point source O that is 90° out of phase with the incoming wave. By imaging the waves in this way we can insert additional optics at point f that can modulate the initial laser pulse and the perturbative pulse independently.

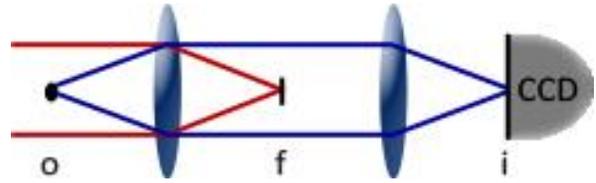


FIG.2 Dark-ground imaging setup: An opaque disk is placed in the center of the focal plane to absorb all unperturbed light. The result is that only the small the diffracted beam is imaged by the CCD.

The intensity recorded at the CCD in FIG. 1 will be:

$$I(x, y) \approx |E_0(x, y)|^2 \approx I_0(x, y)$$

It wasn't until 1996 that a system was designed and built to extract this atomic density dependent phase modulation due to trapped atoms *in situ* [1]. In this setup, a small opaque disk was placed in center of the focal plane (f) that simply served to absorb the unperturbed plane wave as illustrated in FIG.2. This technique, called dark-ground imaging, results in only the diffracted wave making it to the camera. The intensity recorded at the CCD in FIG 2 will be:

$$I(x, y) \approx \phi^2(x, y)I_0(x, y)$$

The phase can then be directly measured by dividing out the initial laser intensity. This technique produced the first images of an atomic cloud undergoing BEC *in situ* (reprinted here in FIG 3).

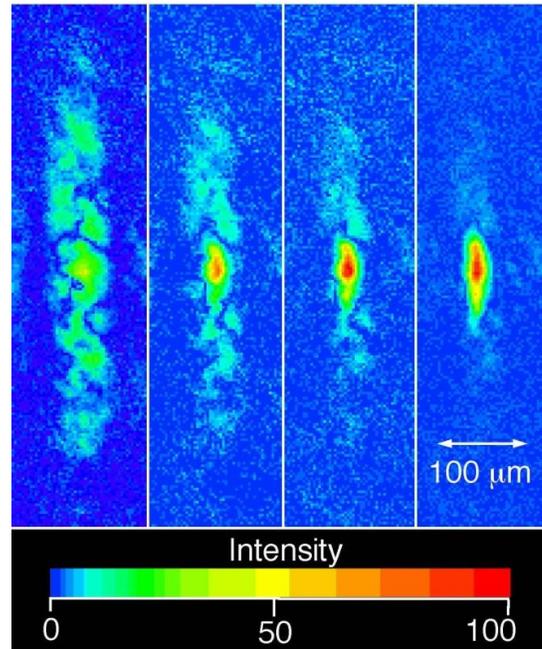


FIG.3 M.R. Andrews *et al* Science 273 (1996)

“Direct observation of BEC of magnetically trapped atoms by dispersive light scattering....The clouds have condensate fractions that increase from close to 0% (left) to almost 100% (right)...The signal for the normal component is rather weak and interferes with the speckle pattern of stray laser light, giving it a patchy appearance.”

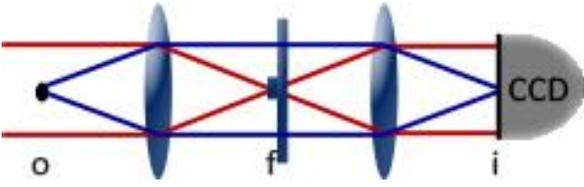


FIG.4 Phase-contrast imaging setup: A pane of glass is inserted into the focal plane (f) with a small raised (depressed) area in the center causing a retardation (advancement) of the phase of the unperturbed plane wave with respect to the diffracted wave. The retardation (advancement) is designed to be a quarter of the free-space wavelength resulting in the constructive (destructive) interference of the two waves.

Dark-ground imaging requires the measurement of very low intensities, resulting in a poor signal to noise ratio and is therefore very sensitive to small fluctuations in background light and optical aberrations. Utilizing the approximation we made above, we can model the phase modulated light as a superposition of two waves out of phase by an optical path length of $\lambda_0/4$, where λ_0 is the free space wavelength [6]. Recognizing this we can build the setup illustrated in FIG. 4. This transforms the electric field found earlier into:

$$\begin{aligned} \frac{E(x, y, z)}{|E_0(x, y)|} &\rightarrow e^{i(\omega t \pm k \frac{\lambda_0}{4})} + \phi(x, y) e^{i(\omega t + \frac{\pi}{2})} \\ &= e^{i(\omega t \pm \frac{\pi}{2})} + \phi(x, y) e^{i(\omega t + \frac{\pi}{2})} \\ &= e^{i\omega t} [1 \pm \phi(x, y)] \end{aligned}$$

The intensity recorded at the CCD in FIG. 4 will be:

$$I(x, y) \approx I_0(x, y) [1 \pm 2\phi + \phi^2]$$

This technique produces a signal that is linear in the phase shift, as opposed to quadratic, and results in images with a much better signal to noise ratio. The formation of BEC was observed using this technique in 1996 [6] leading to the images seen in FIG. 5.

Both methods described above give unprecedented images of *in situ* BEC but rely on the fact that with optical densities > 100 one can detune $\sim 100\Gamma$ and still have enough interaction strength to phase modulate the light. If, on the other hand, we have low optical densities ($OD < 1$) the detuning required to image via phase modulation is on the order of Γ which is often comparable to spatially varying Zeeman shifting. New methods must be developed to image cold and ultra-cold atomic gases in this low optical density limit.

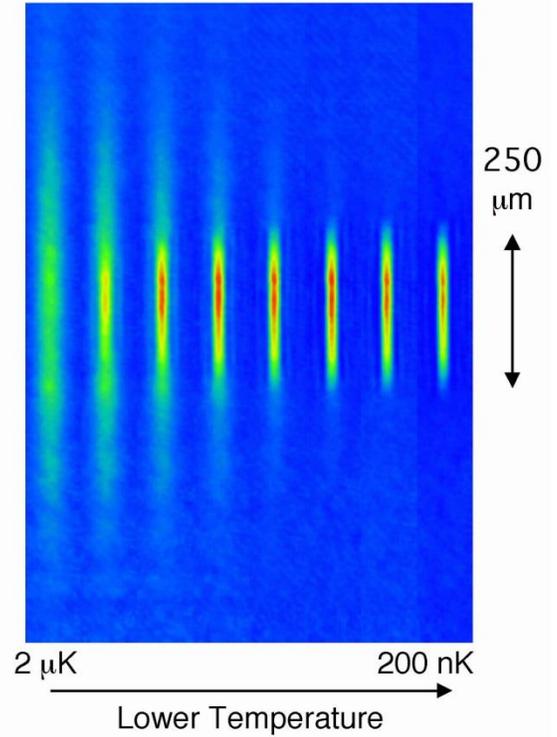


FIG. 5

http://www.rle.mit.edu/cua_pub/ketterle_group/Projects_1996/Dispersive_imaging/non_destructive_imaging.htm
 “Direct observation of the formation of a Bose-Einstein condensate using dispersive light scattering (phase contrast images). The intensity of the scattered light is a measure of the density of atoms (integrated along the line-of-sight). The left picture shows the cloud slightly above the BEC transition temperature. When the temperature was lowered, a dense core formed in the center of the trap - the Bose condensate. Further cooling increased the condensate fraction to close to 100% (right).”

LOW DENSITY *IN SITU* IMAGING

In the high density limit, we removed the spatially varying Zeeman dependence of the laser detuning (Δ) by working in the far detuned limit. In the low density limit we will show that it is possible to remove this same dependence by adding an extra symmetry into the problem and effectively integrating over the Lorentzian absorption profile resulting in a cross section that is independent of magnetic field strength.

As shown earlier, the normalized transmission measurement T , when $I \ll I_{\text{sat}}$, can be written:

$$T = \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \text{Exp} \left[- \int_{-\infty}^{\infty} \frac{\sigma_0 \eta(\vec{r})}{1 + 4(\Delta(\vec{r}, t)/\Gamma)^2} dz \right] dt$$

As we are working in the low optical density limit we can expand the exponential to first order.

$$T \approx 1 - \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \int_{-\infty}^{\infty} \frac{\sigma_0 \eta(\vec{r})}{1 + 4(\Delta(\vec{r}, t)/\Gamma)^2} dz dt$$

$$\Delta(\vec{r}, t) \equiv \omega_{Laser}(t) - [\omega_0 + Z(\vec{r})] = \delta(t) - Z(\vec{r})$$

Where $Z(\vec{r})$ is position dependent Zeeman splitting between the atomic ground and excited state and $\delta(t)$ is the detuning of the laser frequency with respect to bare atomic transition energy ω_0 . Note that now we are allowing δ to be a function of time. The goal will be to choose an appropriate time dependent detuning, such that the transmission becomes invariant under shifts of the effective resonance energy; this can be done by choosing $\delta(t)$ to be linear in t with slope δ' .

$$T \approx 1 - \frac{\Gamma^2 \sigma_0}{\tau} \int_{-\infty}^{\infty} \eta(\vec{r}) \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \frac{1}{\Gamma^2 + 4[\delta' t - Z(r)]^2} dt dz$$

Here we have factored out the natural linewidth Γ and switched the order of integration.

The term inside the inner most integral is simply an offset Lorentzian. It is clear that area under a Lorentzian is independent of a constant offset and therefore the time integration becomes independent of the Zeeman shift under the condition that $\delta' \tau/2$ is large.

To quantify this condition we can make the following substitution

$$u = \delta' t - Z(r) \quad du = \delta' dt$$

$$\begin{aligned} T &\approx 1 - \frac{\Gamma^2 \sigma_0}{\delta' \tau} \int_{-\infty}^{\infty} \eta(\vec{r}) \int_{-\frac{\delta' \tau}{2} - Z(r)}^{\frac{\delta' \tau}{2} - Z(r)} \frac{1}{\Gamma^2 + 4[u]^2} du dz \\ &= 1 - \frac{\Gamma \sigma_0}{2 \delta' \tau} \int_{-\infty}^{\infty} \eta(\vec{r}) \text{ArcTan} \left[\frac{D - 2 Z(r)}{2 \Gamma} \right] dz \\ &\quad - \frac{\Gamma \sigma_0}{2 \delta' \tau} \int_{-\infty}^{\infty} \eta(\vec{r}) \text{ArcTan} \left[\frac{D + 2 Z(r)}{2 \Gamma} \right] dz \end{aligned}$$

where $D \equiv \delta' \tau$ is the range of the detuning sweep. If we want to remove all dependence on the magnetic field strength, we need the arguments within the ArcTan functions to be large. Noting that $Z(r)$ is a varying function in space, the strictest requirements are set at the maximum and minimum Zeeman energies.

Thus far we have assume a linear detuning sweep centered at $\delta(0) = 0$ but of course we are not restricted to that. The optimal sweep would place the Zeeman broadened spectrum centered directly within the detuning

sweep. Using such a sweep effectively shifts the Zeeman interaction, centering it at 0 and yields the strictest requirements at $Z_{max} = -Z_{min} \equiv \Delta Z/2$.

$$\text{if } \text{ArcTan} \left[\frac{D}{2\Gamma} \left(1 - \frac{\Delta Z}{D} \right) \right] \approx \text{ArcTan} \left[\frac{D}{2\Gamma} \right]$$

$$\text{then } T \approx 1 - \frac{\Gamma \sigma_0}{D} \text{ArcTan} \left[\frac{D}{2\Gamma} \right] \eta_{col}$$

The effective cross section is now independent of magnetic field strength and is given by:

$$T \approx 1 - \sigma_{eff} \eta_{col}$$

$$\sigma_{eff} = \frac{\Gamma \sigma_0}{D} \text{ArcTan} \left[\frac{D}{2\Gamma} \right]$$

For a Zeeman broadening of $\sim \Gamma$, a detuning sweep of 5Γ is appropriate. This results in an effective cross section that is ~ 3.5 times smaller than the resonant cross section. This factor can be thought of as the cost of imaging *in situ*.

REFERENCES

- [1] M.R. Andrews *et al* Science **273** (1996)
- [2] M.R. Andrews *et al* Science **275** (1997)
- [3] Atom-photon interactions, Grynberg *et al* (1992)
- [4] Metcalf *et al*, *Laser cooling and Trapping*, (1999)
- [5] D. Steck, *Rubidium 87 D Line Data*, (2003)
- [6] E. Hecht, *Optics* (1987), 2nd ed.
- [7] http://www.rle.mit.edu/cua_pub/ketterle_group